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SIXTH-CENTURY INTUITIVE PROBABILITY: THE STATISTICAL SIGNIFICANCE OF A MIRACLE

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Recently, a number of works have been devoted solely or in part to tracing "precursors" of probabilistic reasoning, including [Rabinovitch 1973; Hacking 1975; Garber & Zabell 1979; Schneider 1977, 1981]. Thus far, one very early instance of such reasoning seems to have escaped notice. Both because of its age and, especially, because of its specific mode of reasoning, it might prove worth investigating.

In Book IV, Chapter 48, of the *History of the Franks*, Gregory of Tours relates the looting of the monastery of Latte. In Thorpe's translation the report runs as follows:

> *A force of hostile troops approached and prepared to cross the river. [...] "This is the monastery of Saint Martin!" cried the monks. "You Francs must not cross over here!" Most of those who heard this were filled with the fear of God and so withdrew. Twenty*

> *of their number, who did not fear God and had no respect for the blessed Saint, climbed into a boat and crossed the river. Driven on by the Devil himself, they slaughtered the monks, damaged the monastery and stole its possessions, which last they made into bundles and piled on their boat. Then they pushed off into the stream, but their keel began to sway to and fro, and they were carried round and round. They had lost their oars, which might have saved them. They tried to reach the bank by pushing the butts of their spears into the bed of the river, but the boat split apart beneath their feet. They were all pierced through by the points of their lances, which they were holding against their bodies; they were all transfixed and were killed by their own javelins.* Only one of them remained unhurt, a man who had rebuked the others for what they were doing. If anyone thinks that this happened by chance, let him consider the fact that one innocent man was saved among so many who were doing evil (emphasis added, J. H.). [Thorpe 1974, 245]

In the critical edition of the Latin text, the sense of the final passage is no different:

> *Unus tantum ex ipsis, qui eos increpabat ne ista committerent, remansit inlaesus. Quod si hoc quis fortuitu evenisse iudicat, cernat, unum insontem plurimis evasisse de noxiis.* [Arndt 1885, 183 f.]

As is the case with most instances of pre-Pascalian probabilistic reasoning discussed by the authors quoted in the Introduction, this piece of reasoning is purely qualitative. There is absolutely no trace of a *calculus*, i.e., a quantitative *calculation*, of probabilities. In this respect, Gregory's argument is on a par with many other pre-Modern precursors of the mathematical theory of probability, as developed in the late 17th century.

Still, inside the field of intuitive and qualitative probabilistic reasoning, Gregory's argument belongs to a *genre* of its own, one not mentioned by any of the above-mentioned authors (with one partial exception; cf. below). It constitutes as much a *test of significance* as can be achieved inside the qualitative framework: it compares two hypotheses, one mentioned explicitly ("by chance" [*fortuitu*]), the other given only implicitly ("not by chance," that is, by Divine Providence, by miracle), and gives the intuitive, qualitative evidence that the *fortuitu* hypothesis must be rejected ("one innocent man ... among so many" [*unum insontem plurimis*]).

Qualitative as this is, it is still closer to mathematics than the arguments on probable truth advanced by the Ancient Sceptics (cf. [Schneider 1977, 176-178]). It is no less mathematical than a partly parallel discussion from the Talmud related by Rabinovitch [1], and it is comparable to a passage from Cardan's autobiography, according to which

> *in a fair game at hazards only three spots* [*with three dice*] *is a natural occurrence, and deserves to be so deemed; and even when they come up the same way for a second time, if the throw be repeated. If the third and fourth plays are the same, surely there is occasion for suspicion on the part of a prudent man.* [Stoner 1930, 150] [2]

Cardan wrote in 1575. Gregory's argument is dated almost a thousand years earlier, to c. A.D. 590 [3]. Without diminishing Gregory's intellectual greatness, it is possible that he may have been less than completely original in this case. After all, A.D. 590 was not a year marked in the West by significant traces of mathematical activity of any kind. It fell within a decade of the midpoint between the death of Boethius (A.D. 524) and the birth of Bede (A.D. 672/673), i.e., between the final and feeble revival of Ancient mathematics and the first modest beginnings of medieval mathematics in the West. Furthermore, instruction in a bishop's *familia* (the bishop's household, where he had to instruct young boys as future clerks) in Merovingian Gaul was hardly a place where even qualitative mathematics was to be learned (see [Riché 1979, 19-41, esp. p. 40]). Gregory received his only formal education in the episcopal *familia*. Later in life he read Latin letters (see [Riché 1979, 20; Thorpe 1974, 8 f.]), which certainly helped to raise his level as a humanist, but which can hardly have taught him much about mathematics. Consequently, it would not be surprising if antecedents for Gregory's "test" of the significance of a miracle were to be found in the literature with which he was familiar. One place to look might be in patristic writings, especially, perhaps, the apologetic fathers, to whom such a test might have seemed apropos.

On the other hand, the fact that such an argument was made at all by the highly intelligent but mathematically illiterate Gregory may be taken as evidence that statistical intuitions are not--and were not--necessarily dependent on mathematical training; whether Gregory borrowed the argument or invented it himself, he understood it. His understanding, however, had little to do with formal mathematical knowledge. Instead, it was another instance of his general reflective and critical approach to the world. Gregory's modest and absolutely inef-

fectual contribution to the prehistory of statistical theory suggests that only part of the truth on "the emergence of probability" will be found by the search for definite intellectual currents which have given rise to the modern concept of probability by being integrated with each other.

NOTES

1. According to a case raised in the Talmud, a widow "married her brother-in-law before the required three-month waiting-peroid had elapsed, and although at three months after her husband's death there were no external signs of pregnancy, she gave birth to a child scarcely six months later, i.e., hardly nine months after her first husband's death. [...] [The case leads to the following] Talmudic argument: 'The majority of women bear [a child] only after nine months' pregnancy ... and for most women who bear at nine months, pregnancy is already recognizable at one third term, but this one, since it was unrecognized at one third term, the majority is weakened.' Rashi [an 11th century commentator] adds: 'You cannot put her into the majority of women, but it is a doubt whether she belongs to the minority or the majority'" [Rabinovitch 1973, 59-60; internal quotations are Rabinovitch's quotations from the Talmud and from Rashi].

2. Incidentally, this passage stands as an argument by analogy when Cardan argues from a series of strange coincidences that his life has been guided by a supernatural providence. So, the coupling between the Ancient understanding of *probabilitas* and the games of chance, definitely achieved in the 17th century, is already on its way by 1575.

3. The chapter does not appear in the early B-version of six books and so belongs to the revision of the work which Gregory made late in life (see [Arndt 1885, 23, 183; Omont 1886, xii, 138]).

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SOME COMMENTS ON RIEMANN'S CONTRIBUTIONS TO DIFFERENTIAL GEOMETRY

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In a recent article in this journal E. Portnoy [1982] gave an interesting assessment of Riemann's contribution to differential geometry. In the present note I would like to comment briefly on some material which was not mentioned in her discussion.

Although her article includes a number of references, it neglects to mention what is undoubtedly the most profound and interesting version of Riemann's probationary lecture. This is [Riemann 1868/1919], which features a detailed commentary by Hermann Weyl. In the third (and final) edition of this forty-eight page monograph more than half of the contents is taken up